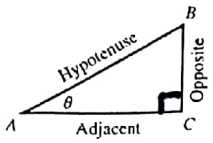


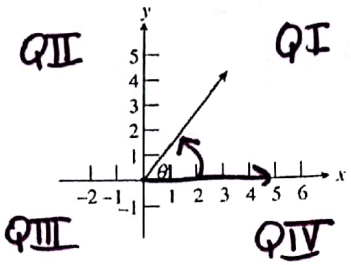


FIGURE 4.7 An acute angle θ in *standard position*, with one ray along the positive x-axis and the other extending into the first quadrant.

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trigonometric functions of the angle θ are defined as follows.

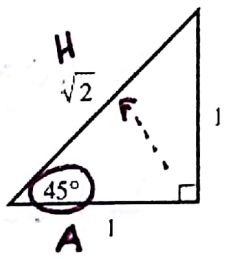
$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

→ No decimal answers! Simplify Radicals

1) Find the value of all six trigonometric functions for a 45-degree angle

$\frac{O}{H} = \sin 45° = \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\frac{H}{O} = \csc 45° = \sqrt{2}$



$\frac{A}{H} = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\frac{H}{A}$ $\sec 45° = \sqrt{2}$

$\frac{O}{A} = \tan 45° = \frac{1}{1} = 1$ $\frac{A}{O}$ $\cot 45° = 1$

FIGURE 4.9 An isosceles right triangle. (Example 1)

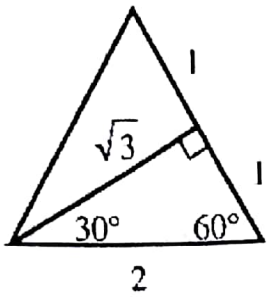
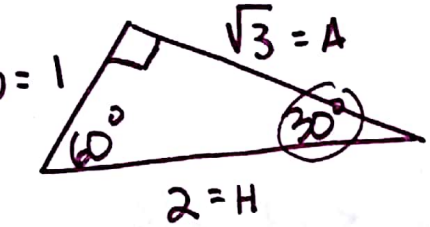


FIGURE 4.10 An altitude to any side of an equilateral triangle creates two congruent 30°–60°–90° triangles. If each side of the equilateral triangle has length 2, then the two 30°–60°–90° triangles have sides of length 2, 1, and $\sqrt{3}$. (Example 2)

2) Find the value of all six trigonometric functions for a 30-degree angle

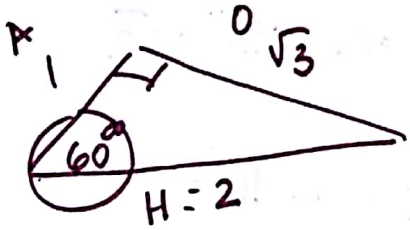


$\sin 30 = \frac{1}{2}$ $\quad \csc 30 = 2$

$\cos 30 = \frac{\sqrt{3}}{2}$ $\quad \sec 30 = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan 30 = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad \cot 30 = \sqrt{3}$

3) Find the value of all six trigonometric functions for a 60-degree angle

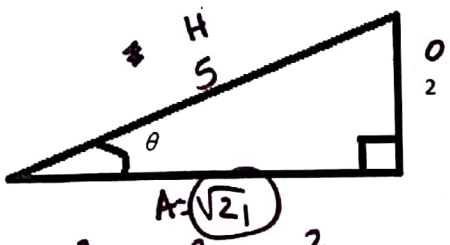


$\sin 60 = \frac{\sqrt{3}}{2}$ $\quad \csc 60 = \frac{2\sqrt{3}}{3}$

$\cos 60 = \frac{1}{2}$ $\quad \sec 60 = 2$

$\tan 60 = \sqrt{3}$ $\quad \cot 60 = \frac{\sqrt{3}}{3}$

4) Find the six trigonometric functions of θ in the figure.



$a^2 + A^2 = 5^2$

$4 + A^2 = 25$

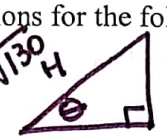
$\sin\theta = \frac{2}{5}$ $\quad \csc\theta = \frac{5}{2}$

$\cos\theta = \frac{\sqrt{21}}{5}$ $\quad \sec\theta = \frac{5}{\sqrt{21}}$

$\tan\theta = \frac{2}{\sqrt{21}}$ $\quad \cot\theta = \frac{\sqrt{21}}{2}$

Assume that θ is an acute angle in a right triangle satisfying the given conditions. Evaluate the remaining trigonometric functions for the following problems.

a) $\cot\theta = \frac{11}{3} \leftarrow \frac{A}{O}$

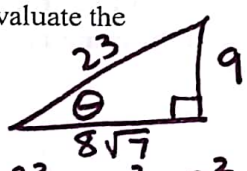


$\sin\theta = \frac{3}{\sqrt{130}}$ $\quad \csc\theta = \frac{\sqrt{130}}{3}$ $\quad 3^2+11^2 = H^2$

$\cos\theta = \frac{11}{\sqrt{130}}$ $\quad \sec\theta = \frac{\sqrt{130}}{11}$ $\quad 9+121 = H^2$

$\tan\theta = \frac{3}{11}$ $\quad \cot\theta = \frac{11}{3}$ $\quad \sqrt{130} = \sqrt{H^2}$

b) $\csc\theta = \frac{23}{9} \leftarrow \frac{H}{O}$



$\sin\theta = \frac{9}{23}$ $\quad \csc\theta = \frac{23}{9}$ $\quad x^2+9^2 = 23^2$

$\cos\theta = \frac{8\sqrt{7}}{23}$ $\quad \sec\theta = \frac{23}{8\sqrt{7}}$ $\quad x^2+81 = 529$

$\tan\theta = \frac{9}{8\sqrt{7}}$ $\quad \cot\theta = \frac{8\sqrt{7}}{9}$ $\quad \sqrt{x^2} = \sqrt{448}$

$\sqrt{64}\sqrt{7}$

$8\sqrt{7}$

6)